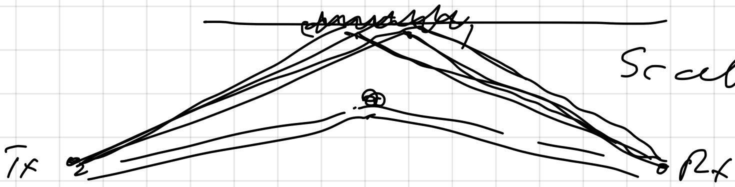
Wireless Receivers

30.10.2020

Multipath Channels (Frequency selective channels)



Scattering → multiple paths
all have diff. but
similar length
$\sim 1 \cdot \lambda$

$$y(t) = \sum_{n=1}^{N} x(t-\tau_n)\, e^{j\phi_n}\, \alpha$$

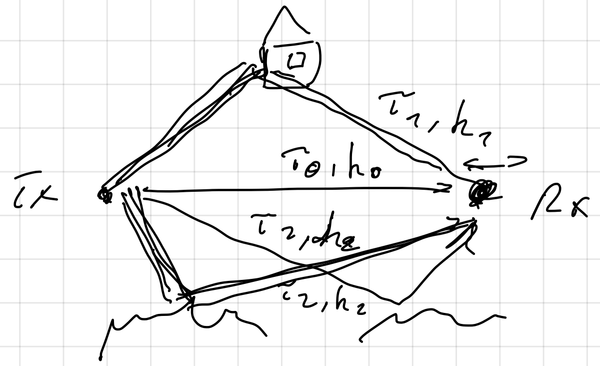
Assumptions: $|\tau_n - \tau_0| \ll T$

diff in arrival time is shorter than the symbol duration

$$x(t-\tau_n) \approx x(t-\underline{\tau_0}) \qquad \tau_0 = 0$$

$$y(t) = x(t) \cdot \underbrace{\sum_{n=1}^{N} e^{j\phi_n} \frac{1}{\sqrt{N}}}_{h}$$

Typical scenario



$$|\tau_n - \tau_0| \gtrsim T$$

diff. in path delays exceeds the symbol duration

$$\Rightarrow \underline{x(t-\tau_n)} \neq x(t)$$

Each path experiences scattering or fading

Ind. path gains: τ_n, $h_n \in \mathcal{CN}(0, \bar{h}_\ell)$

change when we move with a block-fading assumption

Channel model: FIR filter

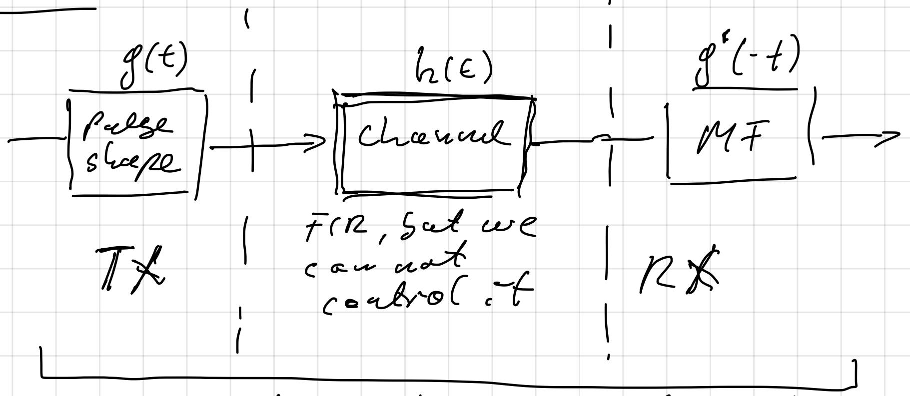
$$\underline{y(t)} = \sum_{l=1}^{L} h_l\, x(t-\tau_l) \rightarrow \text{channel model} \quad x(t) = \delta(t)$$

$$\underbrace{h(t) = \sum_{l=1}^{L} \delta(t-\tau_l)\, h_l}_{\text{FIR-filter}}$$

Multipath channel model

$$x(t) \longrightarrow \boxed{\underset{\text{FIR}}{\overset{\text{channel}}{h(\tau)}}} \longrightarrow y(t)$$

Complete System



Combine these effective channel

$$h_e(\tau) = g(\tau) \circledast h(\tau) \circledast g^*(-\tau)$$

How to characterize $h(\tau)$

- Assumption: Tap delay τ_n are given and fixed
- Strength of the taps: Rayleigh fading

Characterize random impulse response with autocorr fct.

$$R_h(\Delta\tau) = \mathcal{E}\left\{ h(\tau)\, h^*(\tau-\Delta\tau) \right\}$$

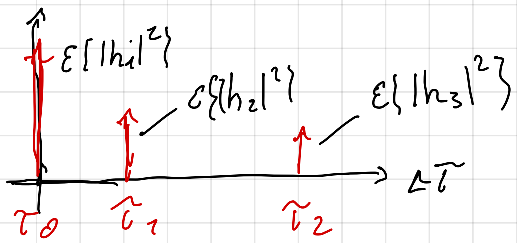
$$= \mathcal{E}\left\{ \sum_{l=1}^{N} h_l\, \delta(\tau-\tau_l) \sum_{l'=1}^{L} h^*_{l'}\, \delta(\tau-\tau_{l'}-\Delta\tau) \right\}$$

if h_l and $h_{l'}$ for $l=l'$ are uncorrelated

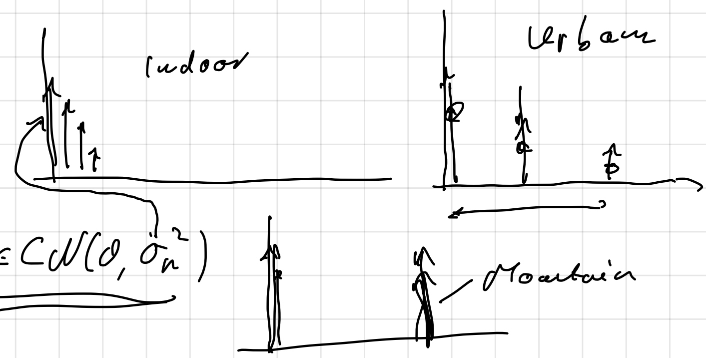
$$\mathcal{E}\{h_l h_{l'}\} = 0 \text{ for } l \neq l'$$

$$= \sum \mathcal{E}\{|h_l|^2\}\, \delta(\tau_l - \Delta\tau) = \begin{cases} \mathcal{E}\{|h_l|^2\} & , \Delta\tau = \tau_l \\ 0 & , 0 \end{cases}$$

Power-delay-Profile



Example: PDPs



$h_n \in \mathcal{CN}(0, \sigma_n^2)$

Receiver Impact of Multipath

$$s(t) \to \boxed{g(\tau)} \to \boxed{h(\tau)} \to \boxed{g^*(-\tau)} \xrightarrow{z(t)} \overset{T}{\text{/}} \to$$

$$s_k \;\text{/}\; \xrightarrow{s(t)} \boxed{h_e(\tau)} \xrightarrow{z(t)} \overset{kT}{\text{/}} \to z_n$$

Sampling @ $n\cdot T$: $z_n = z(nT)$ $\qquad g_{rc} = g \circledast g^*$

$z_n = z(nT)$: $\qquad$ no channel $h(\tau)=\delta(\tau)$ $\qquad z_n = \sum_{k=-\infty}^{+\infty} s_k\, g_{rc}(n\cdot T - kT)$

g_{rc}: fulfills Nyquist Criterion

$$g(nT) = \begin{cases} 1 & n=0 \\ 0, & o \end{cases}$$

$z_n = s_n \to$ no ISI

Effective channel $\quad h(\tau) \neq \delta(t)$

$$h_{eff}(\tau) = \sum_{l=1}^{L} h_l\, g_{rc}(\tau-\tau_l)$$

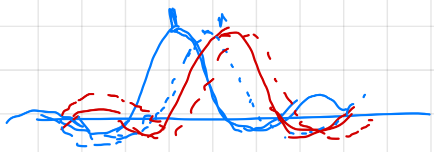
$$z(t) = \sum_{k=-\infty}^{+\infty} s_k \sum_{l=1}^{L} h_l\, g_{rc}(\tau - kT - \tau_l)\Big|_{t=nT}$$

$$= \sum_{l=1}^{L} h_l \sum_{k=-\infty}^{+\infty} s_k\, g_{rc}(\tau-\tau_l-kT)$$

synchronize perfectly: $\tau_1 = 0$

$$z(nT) = h_1 s_n + \sum_{l=2}^{L} s_k\, \underbrace{g_{rc}(nT-\tau_l-kT)}_{\underbrace{g_{rc}((n-k)T-\tau_l)}_{\text{generally } \neq 0}}$$

Inter-symbol Interference



Multipath = Echo

Multipath Channel Estimation

4/11/2022

- Training based

Given: a training (pilot) sequence (known to Tx <u>and</u> Rx)

$$\underline{x} = [x_0, x_1, x_2, \dots x_{N-1}]^T$$

Unknown: channel $\underline{h} = [h_0, h_1, h_2, \dots, h_{L-1}]^T \quad L \leq N$

note: for estimating L-taps we need at least $N \geq L$ training symbols

How to estimate $\underline{h}$ from Rx-vector $y = [y_t, y_{t+1}, \dots, y_{t+P-1}]^T$ $\underline{\underline{P \geq L}}$

1) Write Rx-signal as a function of $\underline{h}$ and $\underline{x}$
↳ Convolution in linear algebra

$$y_t = \sum_{l=0}^{L-1} h_l x_{t-l} \qquad \text{consider } t \geq L-1 \rightarrow t-l \geq 0$$

$$y_t = [x_t, x_{t-1}, x_{t-2}, \dots, x_{t-L+1}] \begin{bmatrix} h_0 \\ h_1 \\ \vdots \\ h_{L-1} \end{bmatrix}$$

for $t = L-1 \dots 2L-2$

$$\underbrace{\begin{bmatrix} x_{L-1}, & x_{L-2}, & x_{L-3}, & \dots, & x_0 \\ x_L, & x_{L-1}, & x_{L-2}, & \dots, & x_1 \\ x_{L+1}, & x_L, & x_{L-1}, & \dots, & x_2 \\ & & \vdots & & \\ x_{2L-2}, & x_{2L-3}, & x_{2L-4}, & \dots, & x_{L-1} \end{bmatrix}}_{\underline{\underline{X}} \in \mathbb{C}^{L \times L}} \underbrace{\begin{bmatrix} h_0 \\ h_1 \\ \vdots \\ h_{L-1} \end{bmatrix}}_{\underline{h}} = \underbrace{\begin{bmatrix} y_{L-1} \\ y_L \\ \vdots \\ y_{2L-2} \end{bmatrix}}_{y} \qquad y = \underline{\underline{X}}\, \underline{h}$$

2) Solve for $\underline{h}$ for $t = L-1, \dots, 2L-2$: $\underline{h} = \underline{\underline{X}}^{-1} y$

for $t = L-1, \dots, L+P-2$ with $P > L$: $\underline{h} = (\underline{\underline{X}}^H \underline{\underline{X}})^{-1} \underline{\underline{X}}^H y$